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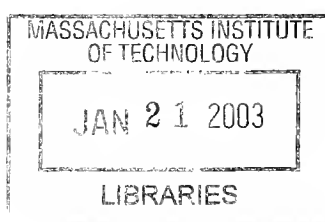
**OPTIMAL TAXATION AND
THE LE CHATELIER PRINCIPLE**

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Abstract

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It is a natural presumption that there should be less distorting taxation when there are more decisions based on the prices distorted by taxation. This note shows the need for an additional assumption in order to reach the conclusion. We consider a competitive model with one consumption good, labor, and human capital. To represent a difference in the number of decisions based on a price, we contrast the situation where human capital is chosen with that where the human capital level is a parameter, using the Le Chatelier principle. Thus, the result is relevant for considering a temporary tax, which just applies to one cohort already in mid-life, with a permanent tax, which will apply to future cohorts over their entire lives. While the Le Chatelier principle signs the difference in substitution effects between the two models, there are other terms that are relevant as well. The assumption that the income derivative of human capital is small relative to its substitution effect is sufficient to sign the response of social welfare to wage taxation with fixed human capital at the value of the optimal wage tax with human capital chosen, thereby giving the presumptive result.

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1 Introduction

It is a natural presumption that there should be less distorting taxation when there are more decisions based on the prices distorted by taxation. This note shows the need for an additional assumption in order to reach the conclusion. We consider a competitive model with one consumption good, labor, and human capital. To represent a difference in the number of decisions based on a price, we contrast the situation where human capital is chosen with that where the human capital level is a parameter, using the Le Chatelier principle. Thus, the result is relevant for considering a temporary tax, which just applies to one cohort already in mid-life, with a permanent tax, which will apply to future cohorts over their entire lives. While the Le Chatelier principle signs the difference in substitution effects between the two models, there are other terms that are relevant as well. The assumption that the income derivative of human capital is small relative to its substitution effect is sufficient to sign the response of social

welfare to wage taxation with fixed human capital at the value of the optimal wage tax with human capital chosen, thereby giving the presumptive result. As is often the case with optimal tax analyses, the presence of income effects complicates the analysis (contrast, e. g., Mirrlees, 1971 with Diamond, 1998).¹

2 Individual Choice

Assuming linear taxation, we can consider the choice problem²

$$\text{Max}_{x,h} u(wxh + b, x, h) \quad x \geq 0, h \geq 1 \quad (1)$$

where w is the net of tax wage, x the hours of labor supply, h the level of human capital investment, and b the level of lump sum income. We assume that u is strictly concave and that the problem has an interior maximum at relevant parameters. For this interpretation, we naturally have $u_1 > 0$, $u_2 < 0$, $u_3 < 0$, with the latter condition reflecting the cost of increasing human capital (earnings per hour of labor supply).³ This choice problem yields optimal choice

¹For other comparative statics results on optimal taxation, see Helpman and Sadka (1978).

²For a more general formulation of the interaction between human capital investments and the return to labor supply, we could denote by $E(x, h)$ the (gross of tax) level of earnings after a human capital investment of h and a labor supply of x hours. Since human capital and the return to working can interact in a rich variety of ways, it might be instructive to explore such a model.

³For example, consider three cases where human capital investment has only financial costs, only adds to the disutility of labor (for example by taking time) and only has an additive utility cost.

$$u(wxh + b, x, h) = f(wxh + b - c(h), x)$$

$$u(wxh + b, x, h) = f(wxh + b, x + c(h))$$

$$u(wxh + b, x, h) = f(wxh + b, x) - c(h)$$

functions $x(w, b)$ and $h(w, b)$. We denote the wage, income and compensated wage derivatives by x_w , x_b , and x_c ; h_w , h_b , and h_c . If h is held constant at some level, the optimal choice of x depends on w , b , and h , $\tilde{x}(w, b, h)$. We are particularly interested in \tilde{x} evaluated at $h(w, b)$. We denote the derivatives of \tilde{x} similarly to those of x .

It is useful to rewrite the problem in terms of effective labor, which we denote by z , $z = xh$.

$$\text{Max}_{z, h} u(wz + b, z/h, h) \quad (2)$$

$$z \geq 0, h \geq 1$$

The point of this version comes from the fact that it is z , not either x , or h alone, that gives the impact of a change in the net wage on utility. Thus the compensated own derivative of z is positive, while there is not a necessary sign condition for either x or h .

Given the structure of the individual choice problems, the Slutsky relations take the form

$$x_c = x_w - xhx_b \quad (3)$$

$$h_c = h_w - xhh_b \quad (4)$$

$$z_c = z_w - zz_b \geq 0 \quad (5)$$

$$\tilde{x}_c = \tilde{x}_w - xh\tilde{x}_b \geq 0 \quad (6)$$

As compensated own derivatives of a bad, we also know that $\tilde{x}_c \geq 0$, $z_c \geq 0$ or

with the assumptions $c(1) = 0$, $c'(h) > 0$, $c''(h) > 0$.

$$x_ch + xh_c \geq 0.$$

We denote the indirect utility functions as $v(w, b)$ and $\tilde{v}(w, b, h)$. By the envelope condition, we have

$$v_w(w, b) = \tilde{v}_w(w, b, h(w, b)); v_b(w, b) = \tilde{v}_b(w, b, h(w, b)). \quad (7)$$

Moreover, the demand derivatives of the two problems satisfy the relationships

$$x_w - \tilde{x}_w = \tilde{x}_h h_w \quad (8)$$

$$x_b - \tilde{x}_b = \tilde{x}_h h_b \quad (9)$$

Note that these expressions can have either sign. That is, human capital may be (but is not necessarily) a normal good ($h_b \leq 0$) and human capital may be increasing or decreasing in the wage.⁴ Also, labor supply can be increasing or decreasing in human capital, just as it may be increasing or decreasing in the wage.

Given the structure of the individual choice problem, the Slutsky relation gives us the condition

$$x_w - \tilde{x}_w = x_c - \tilde{x}_c + xh(x_b - \tilde{x}_b) = x_c - \tilde{x}_c + xh\tilde{x}_h h_b \quad (10)$$

The familiar Le Chatelier Principle (Samuelson (1947)), exposited in Hatta

⁴Note that there is no liquidity constraint in this problem, a source of a positive income derivative of human capital for many people.

(1987), Milgrom and Roberts (1996), and Silberberg (1974)) tells us that for $h = h(w, b)$,

$$z_c(w, b) - \tilde{z}_c(w, b, h) \geq 0 \quad (11)$$

or, using the relationship between effective labor and hours of labor and human capital

$$h(x_c - \tilde{x}_c) + xh_c \geq 0 \quad (12)$$

However, signing the difference in compensated derivatives is not sufficient in general to sign the difference in Marshallian derivatives.

3 Ramsey Taxation

We turn now to a formal analysis of the Ramsey economy. If the revenue needs of the government were fixed, then the level of tax to finance them would be the same in the two economies we are considering - the change in effective labor supply derivatives would not change the result. In order to show the role of elasticities, we assume that the government is using the revenue to finance a public expenditure which has an additive impact on social welfare. Thus we are considering the role of the elasticity on the level of public expenditure, related to the question explored in Atkinson and Stern (1974).

We assume a linear tax on wage income. We are implicitly assuming that h is not observable or not independently taxable for some other reason. For simplicity, we assume a linear technology with a marginal product m of effective

labor. We denote by $g(e)$ the (additive) contribution to welfare of public good expenditure e , with expenditure equal to net tax revenue. Starting with the case of a variable level of human capital we write social welfare as the sum of individual utility (written as a function of the after-tax wage and the level of lump sum income, which is constrained to be zero for the Ramsey problem) and the social value of government expenditures. Then social welfare is written as:

$$\begin{aligned}
 V(w, b) &= v(w, b) + g(e) \\
 \text{where } (w - m)x(w, b)h(w, b) + b + e &= 0 \\
 b &= 0
 \end{aligned} \tag{13}$$

Differentiating V and using Roy's identity, we have the condition for the optimal net-of-tax wage (and so the tax rate):

$$\begin{aligned}
 V_w(w^*, 0) &= -xhv_b + \frac{de}{dw}g' \\
 &= -xhv_b - g'[hx + (w - m)(hx_w + xh_w)] = 0
 \end{aligned} \tag{14}$$

We now fix the human capital level at its optimal level, $h(w^*, 0)$ and define social welfare as the restricted indirect utility function plus the value of public

good expenditure.

$$\begin{aligned}
\tilde{V}(w, b, h(w^*, 0)) &= \tilde{v}(w, b, h(w^*, 0)) + g(e) \\
\text{where } (w - m)\tilde{x}(w, b, h(w^*, 0))h(w^*, 0) + b + e &= 0 \\
b &= 0
\end{aligned} \tag{15}$$

We want to calculate the derivative of social welfare with respect to the net-of-tax wage, evaluated at the optimal wage level for the problem with variable human capital:

$$\begin{aligned}
\tilde{V}_w(w^*, 0, h(w^*, 0)) &= \tilde{v}_w(w, b, h(w^*, 0)) + \frac{de}{dw}g' \\
&= -xhv_b - g'[hx + (w - m)(h\tilde{x}_w)]
\end{aligned} \tag{16}$$

Subtracting (14) from (16) and using the Slutsky equation, we have

$$\begin{aligned}
\tilde{V}_w(w^*, 0, h(w^*, 0)) &= g'(w - m)(h(x_w - \tilde{x}_w) + xh_w) \\
&= g'(w - m)(h(x_c - \tilde{x}_c) + hxh(x_b - \tilde{x}_b) + xh_c + xhxh_b) \\
&= g'(w - m)(h(x_c - \tilde{x}_c) + hxh\tilde{x}_h h_b + xh_c + xhxh_b) \\
&= g'(w - m)(h(x_c - \tilde{x}_c) + xh_c + hx(h\tilde{x}_h + x)h_b)
\end{aligned} \tag{17}$$

We are looking for sufficient conditions for this derivative to be negative, representing a local gain from increasing the tax (lowering the net-of-tax wage). Since we have a positive tax, $w < m$ (assuming positive public expenditures)

and the first and second terms in the last parenthesis on the right hand side add to a positive from the Le Chatelier Principle, (12), a sufficient condition for the presumptive result is $(h\tilde{x}_h + x)h_b \geq 0$. However, we would expect that plausibly effective labor supply in the constrained problem is increasing in human capital, $(h\tilde{x}_h + x) \geq 0$, (just as we expect it to be increasing in the wage) and plausibly human capital is a normal good, $h_b < 0$. This would result in a negative sign. Thus a sufficient condition is that the income derivative of human capital be small.

Intuitively, the issue is that a large income effect changes the payoff from taxation.

4 Optimal Taxation

The Ramsey setting permits analysis of the presumption of less distorting taxation to finance public expenditures when there are fewer decisions. If distorting taxes are used to finance both public expenditures and income redistribution, then we can examine the impact on both of them. For this purpose, we consider the many person optimal tax problem with linear taxation.

Starting with the case of a variable level of human capital we write the sum of individual utilities as a function of the after-tax wage and the level of lump sum income. We denote by $g(e)$ the contribution to welfare of public good

expenditure e , which is equal to net tax revenue. Social welfare is then

$$V(w, b) = \sum (v^i(w, b)) + g(e) \quad (18)$$

$$\text{where } (w - m) \sum (x^i(w, b)h^i(w, b)) + b \sum (1) + e = 0$$

Differentiating V and using Roy's identity, we have

$$\begin{aligned} V_w &= - \sum (x^i h^i v_b^i) + \frac{de}{dw} g' \\ &= - \sum (x^i h^i v_b^i) - g' \left[\sum (h^i x^i) + (w - m) \sum (h^i x_w^i + x^i h_w^i) \right] \\ V_b &= \sum (v_b^i) - g' \left[\sum (1) + (w - m) \sum (h^i x_b^i + x^i h_b^i) \right] \end{aligned} \quad (19)$$

At the optimum (w^*, b^*) , we have $V_w(w^*, b^*)$ and $V_b(w^*, b^*)$ equal to zero.

We now fix the human capital level of each individual at its optimal level, $h^i(w^*, b^*)$ and evaluate the derivatives of the sum of restricted indirect utility functions plus the value of public goods expenditures. We can write the derivatives as

$$\begin{aligned} \tilde{V}_w &= - \sum (x^i h^i v_b^i) + \frac{de}{dw} g' \\ &= - \sum (x^i h^i v_b^i) - g' \left[\sum (h^i x^i) + (w - m) \sum (h^i \tilde{x}_w^i) \right] \\ \tilde{V}_b &= \sum (v_b^i) - g' \left[\sum (1) + (w - m) \sum (h^i \tilde{x}_b^i) \right] \end{aligned} \quad (20)$$

Subtracting (19) from (20) and using the Slutsky equation, we have

$$\begin{aligned}\tilde{V}_w(w^*, b^*) &= g'(w - m) \sum (h^i(x_w^i - \tilde{x}_w^i) + x^i h_w^i) \\ &= g'(w - m) \sum (h^i(x_c^i - \tilde{x}_c^i) + x^i h_c^i + h^i x^i (h^i \tilde{x}_h + x^i) h_b^i)\end{aligned}\tag{21}$$

This leads to the same analysis of the welfare effect of increased wage taxation as with the Ramsey case.

Turning to the guaranteed income provided, similarly, we have

$$\begin{aligned}\tilde{V}_b(w^*, b^*) &= g'(w - m) \sum (h^i(x_b^i - \tilde{x}_b^i) + x^i h_b^i) \\ &= g'(w - m) \sum ((h^i \tilde{x}_h^i + x^i) h_b^i).\end{aligned}\tag{22}$$

Thus a sufficient condition for the presumptive result that less guaranteed income is provided, $\tilde{V}_b(w^*, b^*) < 0$ is the presence of wage taxation, $w < m$, and $\sum ((h^i \tilde{x}_h^i + x^i) h_b^i) < 0$. We would expect that effective labor supply in the constrained problem is increasing in human capital, $(h \tilde{x}_h + x) \geq 0$, and human capital is a normal good, $h_b < 0$. This would result in the presumed negative sign. Considering a less plausible case, if the income derivative is small but of the implausible sign, we still get an increase in the wage tax when human capital is fixed, but the increased revenue from the increase in the wage tax is used to finance the additive public good, as is the saved revenue from lowering the lump sum income.

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